Equivalent Circuit Parameters of an Aperture Coupled Open Resonator Cavity

R. K. Mongia, *Member, IEEE,* and R. K. Arora, *Senior Member, IEEE*

In this paper, we use modified form of Bethe's small hole coupling theory to compute equivalent circuit parameters of an aperture coupled open resonator cavity. The open resonator cavity is composed of spherical mirrors of circular cross section. The cavity is coupled to a rectangular waveguide by means of a common hole in the mirror and the shorted end wall of the rectangular waveguide. Closed form expressions have been obtained for the equivalent circuit parameters. Experiments conducted in the W-band frequency range show good agreement with theory when an experimentally estimated correction to the transmission coefficient is applied for the thickness of the coupling holes.

I. Introduction

OPEN RESONATORS offer unique advantages over closed cavities at higher microwave frequencies and beyond [1]. Among their applications are their use as resonant cavities in gyrotrons [2] and in the determination of electrical properties of dielectric materials [3], [4]. Two popular open resonator configurations are shown in Figs. 1 and 2. The open resonator configuration shown in Fig. 1 comprises spherical mirrors of circular cross section. This configuration has been used in gyrotrons. The configuration shown in Fig. 2 is a variant of the configuration shown in Fig. 1 where the plane $z' = 0$ is replaced by a conductor and the input and output couplings are in the same mirror. The configuration shown in Fig. 2 is attractive for the use of open resonator cavity for the characterization of dielectric materials because the dielectric sample can be placed directly on the flat surface [3], [5].

The resonant modes of open resonators have been extensively studied in the past by using both the scalar theory [6], [7] and the rigorous vector theory [4], [8]. However, relatively little effort has been devoted to the study of coupling to open resonators. At microwave and millimeter-wave frequencies, a popular scheme of coupling is by means of a small hole in the mirror and the shorted end wall of a rectangular waveguide as shown in Figs. 1 and 2. The overall cavity operates in the transmission mode. Due to the open geometry, the coupling holes may also radiate into free space. It has been shown by Cullen that the power radiated by small coupling holes into the free space is lower, by 2–3 orders of magnitude, than the power lost in the imperfect conducting walls of the cavity mirrors [9]. The unloaded Q-factor of the cavity is thus not affected by the direct radiation from coupling holes into the free space.

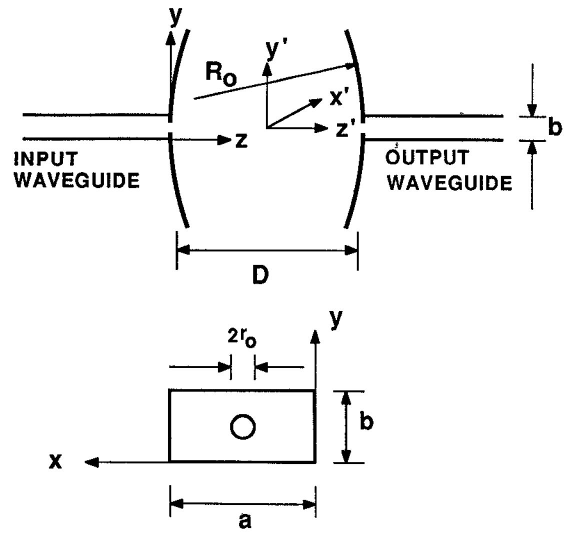


Fig. 1. An open resonator cavity excited by a rectangular waveguide.

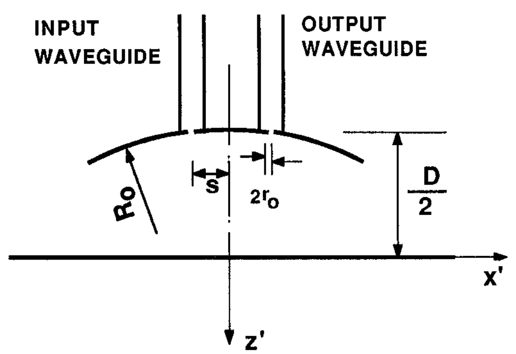


Fig. 2. A plano concave open resonator cavity.

In this paper, we use the modified form of Bethe's small aperture coupling theory as given by Collin [10] to compute the equivalent circuit for the coupling schemes shown in Figs. 1 and 2. It has been noted by Collin that reaction terms have to be added to the original Bethe's small coupling theory to make it consistent and the equivalent circuit to be physically realizable. Finally, some experimental results are also reported and comparison is made with theory.

Manuscript received April 24, 1992; revised September 16, 1992.
R. K. Mongia was with the Department of Electrical Engineering, FAMU/FSU College of Engineering, Tallahassee, Florida. He is presently with Department of Electrical Engineering, University of Ottawa, Ottawa, Ontario, Canada K1N 6N5.
R. K. Arora is with the Department of Electrical Engineering, FAMU/FSU College of Engineering, Tallahassee, FL 32316-2175.
IEEE Log Number 9207416.

0018-9480/93$03.00

II. Theory

We first consider the coupling for configuration as shown in Fig. 1. We assume that a TE_{10} mode is incident in the input rectangular waveguide. For a rectangular waveguide, the normalized mode functions of propagating TE_{10} modes are given as, [10]

$$\begin{aligned} \boldsymbol{E}_{10}^{+} &= \boldsymbol{e}_{10} e^{-j\beta_{10} z}, \\ \boldsymbol{H}_{10+} &= (\boldsymbol{h}_{10} + \boldsymbol{h}_{z10}) e^{-j\beta_{10} z} \\ \boldsymbol{E}_{10-} &= \boldsymbol{e}_{10} e^{j\beta_{10} z}, \\ \boldsymbol{H}_{10-} &= (-\boldsymbol{h}_{10} + \boldsymbol{h}_{z10}) e^{j\beta_{10} z} \end{aligned} \quad (1)$$

In the above equations $\boldsymbol{e}_{10}, \boldsymbol{h}_{10}$ and $\boldsymbol{h}_{z10}$ are given by

$$\begin{aligned} \boldsymbol{e}_{10} &= -jk_o Z_o N \sin(\pi x/a) \boldsymbol{a}_y, \\ \boldsymbol{h}_{10} &= j\beta_{10} N \sin(\pi x/a) \boldsymbol{a}_x \\ \boldsymbol{h}_{z10} &= \frac{N\pi}{a} \cos(\pi x/a) \boldsymbol{a}_z \end{aligned} \quad (2)$$

where

$$N = (-2/abk_o Z_o \beta_{10})^{1/2}$$

Let the amplitude of the incident wave be unity. The incident wave can then be expressed as

$$\boldsymbol{E}_i = \boldsymbol{E}_{10+} \quad (3)$$

The incident wave has no electric field perpendicular to the plane of the coupling hole. Therefore, no electric dipole is induced in the hole. There is only an x-component of the tangential magnetic field, so only an x-directed magnetic dipole is induced. The coupling hole is assumed to be small and of zero thickness. Let $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ be the magnetic dipole moments induced in the input and output holes. According to Collin [10, p. 510]

$$\boldsymbol{M}_1 = \overline{\overline{\alpha}}_m \cdot (\boldsymbol{H}_{g1} + \boldsymbol{H}_{1r} - \boldsymbol{H}_{2r1} - \boldsymbol{H}_{2r2}) \quad (4)$$

where $\overline{\overline{\alpha}}_m$ denotes the dyadic magnetic polarizability of the hole, $\boldsymbol{H}_{g1}$ is the magnetic field due to incident wave in the absence of the coupling hole, and $\boldsymbol{H}_{r1}, \boldsymbol{H}_{2r1}$ and $\boldsymbol{H}_{2r2}$ are the reaction fields. $\boldsymbol{H}_{r1}$ is the dominant mode field produced in the waveguide by the dipole $\boldsymbol{M}_1$. $\boldsymbol{H}_{2r1}$ and $\boldsymbol{H}_{2r2}$ are the dominant fields produced in the resonant cavity by $-\boldsymbol{M}_1$ and $-\boldsymbol{M}_2$. All the field values are calculated at the center of the hole. It may be noted that in the original Bethe's theory, the terms $\boldsymbol{H}_{1r}, \boldsymbol{H}_{2r1}$ and $\boldsymbol{H}_{2r2}$ are not included. In the present case, (4) can be written as

$$M_{1x} = \alpha_m (H_{g1x} + H_{1rx} - H_{2r1x} - H_{2r2x}) \quad (5)$$

where the extra subscript x denotes the x-component of the corresponding field vector. For a circular hole, α_m is related to the radius of the coupling hole r_o by

$$\alpha_m = \frac{4r_{o^3}}{3}$$

We now proceed to calculate different terms of (5).

Evaluation of H_{g1x}

If the hole size is small compared to waveguide dimensions, the tangentialmagnetic field in the absence of coupling hole is twice the incident field. From (1)–(3), H_{g1x} is given by

$$H_{g1x} = 2j\beta_{10} N \quad (6)$$

Evaluation of H_{1rx}

A transverse magnetic dipole M_{1x} placed in an infinite groundplane radiates as a magnetic dipole of double the strength. H_{1rx} term in (5) is thus found as

$$\begin{aligned} H_{1rx} &= (j\omega\mu_o \boldsymbol{H}_{10+} \cdot M_1) \\ &\quad \cdot \boldsymbol{H}_{10-} \cdot a_x = -2j\beta_{10} M_{1x}/ab \end{aligned} \quad (7)$$

Evaluation of H_{2r1x}

To compute the reaction term H_{2rx1}, we first need to normalizemodes of the open resonator. The resonator modes are normalized as

$$\int \boldsymbol{E}_p \cdot \boldsymbol{E}_p \, dv = 1 \quad (8)$$

where $\boldsymbol{E}_p$ denotes the electric field for the pth resonant mode. We are interested in exciting the TEM_{00q} modes of the open resonator. For the TEM_{00q} modes of the open resonator the predominant field components that exist inside the resonator for the assumed excitation are $E_{y'}$ and $H_{x'}$. The expression for electric field inside the resonator for the TEM_{00q} modes can be written as [4]

$$\begin{aligned} \boldsymbol{E}_p &= E_{py} \boldsymbol{a}_{y'} = A_1 \frac{w_o}{w} \exp(-r'^2/w^2) \\ &\quad \cdot \begin{pmatrix} \cos \\ \sin \end{pmatrix} (kz' - \pi + kr'^2/2R) \boldsymbol{a}_{y'} \end{aligned} \quad (9)$$

where r' denotes the axial distance from the axis of the resonator and the cos and sin terms refer, respectively, to symmetrical and anti-symmetrical modes with respect to $z' = 0$ plane. The various terms in (9) are given by

$$\phi = \tan^{-1}(z'/z_o) \quad (10a)$$

$$w_{o^2} = \frac{\lambda_o}{2\pi} \sqrt{\{D(2R_o - D)\}} \quad (10b)$$

$$z_o = \pi w_{o^2}/\lambda_o \quad (10c)$$

$$w^2 = w_{o^2}(1 + z'^2/z_{o^2}) \quad (10d)$$

$$r'^2 = x'^2 + y'^2 \quad (10e)$$

$$R = z'(1 + z_{o^2}/z'^2) \quad (10f)$$

where the symbols have their usual meaning. The second and third terms in the argument of trigonometric function in (9) exist because the wavefront, in general, is not plane. Equations (9) and (10) show that the wavefront is plane in the $z' = 0$ plane but its radius of curvature decreases away from the $z' = 0$ plane and becomes the same as those of the mirrors at the location of the mirrors. If the radius of curvature of mirrors is large, we can assume that the wavefront is plane at all locations inside the resonator. Therefore, the second and third terms in the argument of the trigonometric function can be neglected. Equation (9) then becomes,

$$\boldsymbol{E}_p = E_{py}\boldsymbol{a}_{y'} = A_1 \frac{w_o}{w} \exp \cdot (-r'^2/w^2) \begin{pmatrix} \cos \\ \sin \end{pmatrix} (kz') a_{y'} \quad (11)$$

The above approximation helps us in deriving a closed form expression for the normalization constant A_1. Putting the value of $\boldsymbol{E}_p$ from (11) in (8), we get

$$A_1 = \frac{2}{w_o\sqrt{(\pi D)}} \quad (12)$$

The expression for normalized magnetic field of the resonator is given by [10, p. 386]

$$\boldsymbol{H}_p = \frac{1}{k} \nabla \boldsymbol{x} \boldsymbol{E}_p \quad (13a)$$

In the present case, $\boldsymbol{E}_p$ is given by (11). Therefore,

$$\boldsymbol{H}_p = H_{px}\boldsymbol{a}_{x'} = -A_1 \frac{w_o}{w} \exp \cdot (-r'^2/w^2) \begin{pmatrix} -\sin \\ \cos \end{pmatrix} (kz') \boldsymbol{a}_{x'} \quad (13b)$$

where the upper term $-\sin(kz')$ is to be used for symmetrical modes and the lower one is to be used for anti-symmetrical modes.

Once the normalized fields are known, one can compute the reaction termH_{2rx1}. If the resonator field excited in the resonator by magnetic dipole $-\boldsymbol{M}_1$ is $h_{p1}\boldsymbol{H}_p$, then h_{p1} is given by [10, p. 394]

$$h_{p1} = \frac{-k_o^2 \boldsymbol{H}_p \cdot \boldsymbol{M}_1}{k_{or}^2 - k_o^2\{1 + (1-j)/Q\}} \quad (14)$$

where k_{or} is the free space wavenumber corresponding to the resonant frequency and k_o is the free space wavenumber corresponding to any angular frequency ω close to the resonant frequency. In our case $\boldsymbol{H}_p$ is given by (13), so h_{p1} is computed as

$$h_{p1} = \frac{A_1 k_o^2 M_{1x}}{[k_{or}^2 - k_o^2\{1 + (1-j)/Q\}]} \cdot \frac{w_o}{w_d} \begin{pmatrix} -\sin \\ \cos \end{pmatrix} (-kD/2) \quad (15)$$

where w_d is the width of the Gaussian beam at $z' = -D/2$ and can be calculated using relations (10a)–(10d).

The magnetic field at input hole due to resonator field excited by dipole$-M_{1x}$ is therefore given by multiplying (15) by (13b) and evaluating the resulting term at the center of input hole. Noting that there is an antinode of magnetic field at $z' = \pm d/2, H_{2rx1}$ is found as

$$H_{2rx1} = -\frac{k_o^2 M_{1x} A_1{}^2}{[k_{or}^2 - k_o^2\{1 + (1-j)/Q\}]} \cdot \frac{w_o^2}{w_d^2} = -W_1 M_{1x} \quad (16a)$$

where

$$W_1 = \frac{A_1^2 k_o^2}{[k_{or}^2 - k_o^2\{1 + (1-j)/Q\}]} \cdot \frac{w_o^2}{w_d^2} \quad (16b)$$

Evaluation of H_{2r2x}

Let the magnetic field excited in the resonator due to magnetic dipole $-M_{2x}$ be given by $h_{p2}\boldsymbol{H}_p$. Noting that the width of the beam at $z = D/2$ is also w_d, h_{p2} is computed as

$$h_{p2} = \frac{A_1 k_o^2 M_{2x}}{[k_{or}^2 - k_o^2\{1 + (1-j)/Q\}]} \cdot \frac{w_o}{w_d} \begin{pmatrix} -\sin \\ \cos \end{pmatrix} (kD/2) \quad (17)$$

or, the magnetic field at the input hole due to the resonator field excited by $-M_{2x}$ is given by

$$H_{2rx2} = \pm \frac{k_o^2 M_{2x} A_1{}^2}{[k_{or}^2 - k_o^2\{1 + (1-j)/Q\}]} \cdot \frac{w_o{}^2}{w_d{}^2} = -W_2 M_{2x} \quad (18a)$$

where

$$W_2 = \mp \frac{A_1 k_o^2}{[k_{or}^2 - k_o^2\{1 + (1-j)/Q\}]} \cdot \frac{w_o{}^2}{w_d^2} \quad (18b)$$

In (18a) and (18b), the upper sign is to be used for symmetrical modes and the lower sign is to be used for anti-symmetrical modes.

Once all the terms appearing in (5) have been evaluated, the following equation can be written for the magnetic dipole induced in the input hole

$$M_{1x} = \alpha_m \left[2j\beta_{10}N - \frac{2j\beta_{10}M_{1x}}{ab} + W_1 M_{1x} + W_2 M_{2x} \right] \quad (19)$$

Equation (19) gives one relation between M_{1x} and M_{2x}. To get another relationship between them, we write (3) for the output hole.

Proceeding in a similar manner as above and noting that there is no incident field in the output waveguide, we get the following relationship between M_{1x} and M_{2x}:

$$M_{2x} = \alpha_m \left[\frac{-2j\beta_{10}M_{2x}}{ab} + W_1 M_{2x} + W_2 M_{1x} \right] \quad (20)$$

Using (19) and (20), one can find values for M_{1x} and M_{2x} as

$$M_{1x}\left[1 + \alpha_m(2j\beta_{10}/ab - W_1) - \frac{(\alpha_m W_2)^2}{1 + \alpha_m(2j\beta_{10}/ab - W_1)}\right]$$

$$= 2j\beta_{10}\alpha_m N \quad (21a)$$

$$M_{2x} = \frac{\alpha_m W_2}{1 + \alpha_m(2j\beta_{10}/ab - W_1)} M_{1x} \quad (21b)$$

Input Admittance

The reflected signal in input waveguide is

$$\tau = -1 + b_1 \quad (22)$$

where b_1 is the reflected signal due to dipole M_{1x} placed transversely in metallic plane. Its value is given by

$$b_1 = \frac{j\omega\mu_o}{2} \cdot \boldsymbol{h}_{10} \cdot 2M_{1x}\boldsymbol{a}_x = -k_o Z_o \beta_{10} N M_{1x} \quad (23)$$

Therefore, the reflection coefficient in the input guide is,

$$\tau = -1 - k_o Z_o \beta_{10} N M_{1x} \quad (24)$$

Defining $X = 2\alpha_m\beta_{10}/ab$, and $U = (2j\beta_{10}/ab) - W_1$, one gets after substituting value of M_{1x} from (21a) into the above equation

$$\tau = -1 + \frac{2jX(1 + \alpha_m U)}{(1 + \alpha_m U)^2 - (\alpha_m W_1)^2} \quad (25)$$

The input admittance looking into the resonator is thus given by

$$Y_{in} = \frac{1-\tau}{1+\tau} = \frac{1}{jX} - \frac{\alpha_m W_1}{jX\{1 - \alpha_m W_1/(1 + jX)\}} \quad (26a)$$

Further, defining $K = \alpha_m A_1^2 w_o^2 / w_d^2$, Y_{in} becomes

$$Y_{\text{in}} = \frac{1}{jX} - \frac{K^2\omega}{jX[\omega_{r^2} - \omega^2\{1 + (1-j)/Q_u\}] - K\omega^2/(1 + 1/jX)} \quad (26b)$$

Equivalent Circuit

To represent the above form of input admittance we choose the equivalentcircuit as shown in Fig. 3. The input admittance for this circuit is given by

$$Y_{\text{in}} = -jB - \frac{n^2\omega^2}{j\omega L(\omega_r^2 - \omega^2 - \omega^2/Q_u + j\omega^2/(1 - jB)} \quad (27)$$

where

$$\omega_r^2 = 1/LC, \quad (28a)$$

$$R = \omega_r L/Q_u, \text{ and} \quad (28b)$$

$$\omega L_S = R \quad (28c)$$

It is seen that (26b) and (27) have the same form. Therefore, the equivalent circuit shown in Fig. 3 is suitable for representing the coupling scheme as shown in Fig. 1.

Since $L_S/L = 1/Q_u$ at resonance, L_S can be neglected in comparison to L if $Q_u \gg 1$.

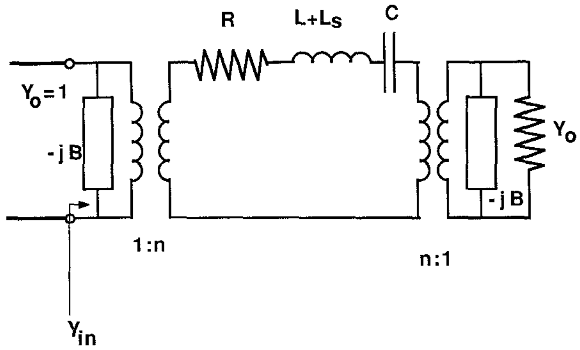


Fig. 3. Equivalent circuit representation ofopen resonator cavities shown in Figs. 1 and 2.

Comparing (26b) with (27), we get

$$B = 1/X = ab/2\beta_{10}\alpha_m \quad (28d)$$

and

$$\frac{n^2}{\omega L} = \frac{K}{X} = \frac{abw_{o^2}}{2\beta_{10}w_{d^2}} A_1^2 \quad (28e)$$

It may be noted that there are three equations (28a), 28(b) and (28e) with four unknowns L, C, R, and n^2. To compute their values, we arbitrarily choose $C = 1$. This gives

$$L = 1/\omega_r^2 \quad (29a)$$

Further, using (28e) we get

$$n^2 = \frac{ab\ w_o^2}{2\beta_{10}\omega_r^2 w_d^2} A_1^2 \quad (29b)$$

Once L is known, the resistance R can be computed using (28b). Further, it is seen from (27) that the real part of $n^2/(1 - jB)$ denotes the external resistance R_e. Its value is found as

$$R_e = \frac{abA_1^2 w_o^2}{\{1 + (ab/2\beta_{10}\alpha_m)^2\} 2\beta_{10} w_d^2 \omega_r} \quad (30)$$

The external Q-factor is thus given by

$$Q_e = \frac{1}{R_e\omega_r C} = \frac{\{1 + (ab/2\beta_{10}\alpha_m)^2\} 2\beta_{10} w_d^2}{abA_1^2 w_o^2} \quad (31)$$

Since the final form of Q_e is very simple and in closed form, one can find the response of the cavity in a simple manner using the equivalent circuit approach.

Transmission Through Cavity

The coupling coefficient is defined as

$$\kappa = Q_u/Q_e \quad (32)$$

If the holes are of same size in both the mirrors, the transmission through the cavity at resonance is given by, [11]

$$T(\omega_r) = \frac{4\kappa^2}{(1 + 2\kappa)^2} \quad (33)$$

One can also compute the transmitted power in the output waveguide using field theory approach by computing the fields excited in the output waveguide by the magnetic dipole M_{2x} whose value is known ((21b)). For the equivalent circuit to be valid, both approaches should lead to the same numerical results.

Coupling for the Configuration of Fig. 2

The analysis can be carried out along similar lines are outlined above for the configuration of Fig. 1. It may be noted that the structure shown in Fig. 2 supports anti-symmetric modes of the structure shown in Fig. 1. In this case the normalization constant A_1 appearing in (11) is found as

$$A_1' = \frac{1}{w_o}\sqrt{(8/\pi D)} \tag{34}$$

The resulting expression obtained for the external Q-factor is the same as given by (31) except that the term A_1 is replaced by $A_1' \exp(-s^2/w_d^2)$ where A_1' is given by (34).

III. Results

In the preceding section, a closed form expression was obtained for the external Q-factor. Its value can be easily computed once the structural parameters are known. For small coupling holes, the term $(ab/2\beta_{10}\alpha_m)^2 \gg 1$. Therefore, as (31) and (32) show, the coupling coefficient κ varies with radius of the coupling hole as r_o^6. Further, using (33), one finds that the transmitted power through cavity varies as r_{o12} for small values of coupling coefficient κ. The transmitted power thus depends very strongly on the hole size.

By putting value of A_1 from (12) in (31), it is seen that for the configuration shown in Fig. 1 the external Q-factor varies as $w_d^2 D$. This is in contrast to the external Q -factor of parallel plate Fabry–Perot resonators in which case the external Q-factor increases linearly with D [12]. In a Fabry–Perot resonator of parallel plate resonators, the unloaded Q-factor also increases linearly with D. Therefore as (32) and (33) show, the transmission through a parallel plate Fabry–Perot cavity remains unchanged as the distance between mirrors is varied. However, with resonators formed of spherical mirrors, this is not the case. The unloaded Q-factor in this case still increases in a linear fashion. However, the external Q factor varies as $w_d^2 D$. Therefore, the power transmitted through an open resonator cavity composed of spherical mirrors would vary with separation. For small values of coupling, the power transmitted thorugh the resonator would vary as w^-4_d with separation as shown by (31)–(33).

IV. Experimental Results

Experiments were conducted at about 94 GHz to determine the coupling for the configuration shown in Fig. 1. The cavity was set up to characterize dielectric materials in the W-band. The mirrors used had a radius of curvature of 330 mm and the mirror radius was 22.5 mm. The radius of the coupling hole was 0.5 mm. This value of radius cannot be termed "small." In our experiments, relatively large-size hole was used to obtain a reasonable level of output power. The thickness (axial length) of the coupling holes was about 0.5–1 mm which caused an appreciable attenuation in the coupled power. Considering that the thick hole acts as a section of below cut-off waveguide, a 1 mm length hole can cause attenuation of about 26 dB at 94 GHz. Due to relatively large coupling hole and finite thickness of coupling holes, we do not expect our theoretical results to match with experiment. The theory presented in this paper is valid for small coupling holes of zero thickness.

TABLE I
Theoretical and Measured Transmission Through an Open Resonator Cavity (a = 2.54 mm, b = 1.27 mm, f_o = 93.6 GHz, R_o = 330 mn, r_o = 0.5 mm)

D (cm)	Q_u	Q_e	Coupling(dB) Theory*	Coupling(dB) Measured*
4.03	3.38×10^4	1.06×10^5	-8.2	-34.6
6.12	5.21×10^4	2.02×10^5	-9.4	-35.8
8.05	5.50×10^4	3.01×10^5	-11.6	-39.0
11.10	6.58×10^4	5.16×10^5	-13.8	-41.6
14.15	5.64×10^4	7.64×10^5	-17.8	-46.1
16.08	2.93×10^4	9.43×10^5	-24.7	-53.6
18.00	2.90×10^4	1.14×10^6	-26.3	-56.5
20.09	2.45×10^4	1.37×10^6	-29.3	-57.8
22.02	2.35×10^4	1.61×10^6	-30.9	-60.1
24.10	1.84×10^4	1.89×10^6	-34.4	-63.0
26.03	1.31×10^4	2.17×10^6	-38.5	-68.4
28.11	0.96×10^4	2.50×10^6	-42.4	-70.0

* The theoretical results are for coupling holes of zero thickness while experimental results shown are for the finite thickness of the coupling holes.

It has been shown that effect of finite thickness of the coupling hole is to cause reduction in coupling. The amount of reduction in coupling depends only on the size and thickness of the hole [13], [14]. Further, it has also been shown by Cohn [13] that the effect of large hole size is to introduce a multiplicative factor in the coupling. This factor also depends only on the size of the hole. We can therefore check our results for the variation of coupling as the spacing between mirrors is varied. The effect of "large" and thick coupling holes can be considered to be the same for all values of spacing between the mirrors.

For theoretical computation, the experimentally measured value of Q-factor was used as the unloaded Q-factor. By increasing the separation between mirrors, the unloaded Q-factor first increases but it starts decreasing after some distance due to diffraction loss. Therefore, measured value of Q-factor is chosen as the unloaded value of Q-factor. The measured values of Q-factor and coupling are shown in Table I for various values of spacing between the mirrors. Strictly speaking, the measured value of Q is the loaded Q (loaded by hole coupling), but in the present case this also represents the unloaded Q-factor because the coupling is very small in all cases as seen from Table I. The theoretical values of coupling are also shown in the same Table. It is seen that, as expected, measured value of coupling is much lower than that predicted by theory mainly because of finite thickness of the coupling

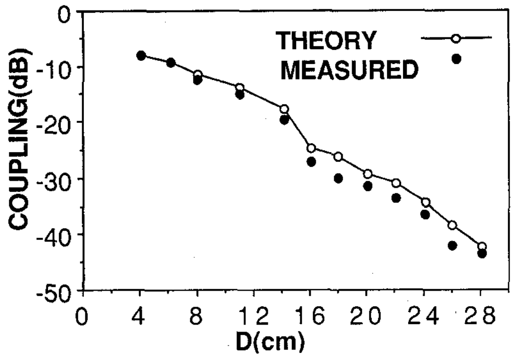


Fig. 4. Comparison of theoretical and experimental results for transmission through cavity shown in Fig. 1. The measured results have been increased by an amount of 26.4 dB which is the effect due to large-size and finite thickness of coupling holes.

holes. For $D = 4$ cm, the theoretically predicted value of coupling is -8.2 dB whereas the measured value is -34.6 dB. We assume that the difference of 26.4 dB between theory and measured values is due to the attenuation caused by finite thickness and relatively large size of the coupling holes. If we add this value of 26.4 dB to all measured values, we find that the agreement between theory and experiment is quite good. In Fig. 4, we plot the measured value of Table 1 increased by an amount of 26.4 dB. The theoretical values from the same Table are also drawn in Fig. 4. It is seen that the agreement between these results is very good for all values of separation 'D' for which measurements were made. These results validate the form of expressions of the theoretical results.

V. Conclusions

In this paper, we use modified form of Bethe's hole coupling theory to compute coupling between a rectangular waveguide and an open resonator through a small hole of zero thickness. Simple closed form expressions have been obtained for the external Q-factor. The results reported in this paper are believed to be useful for designing aperture couplings for devices such as open resonator power combiners, gyrotrons etc.

References

[1] R. N. Clarke and C. B. Rosenberg, "Fabry-Perot and open resonators at microwave and millimeter wave frequencies 2-300 GHz," *J. Phys. E,* vol. 15, pp. 9–24, 1982.

[2] A. W. Fliflet, T. A. Hargreaves, W. M. Manheimer, R. P. Fischer, and M. L. Barsanti, "Operation of quasioptical gyrotron with variable mirror separation," *Phys. Rev. Lett.,* vol. 62, pp. 2664–2667, 1989.

[3] R. J. Cook, R. G. Jones, and C. B. Rosenberg, "Comparison of cavity and open-resonator measurements of permittivity and loss angle at 35 GHz," *IEEE Trans. Instr. Meas.,* vol. IM-23, pp. 438–442, Dec. 1974.

[4] A. L. Cullen, "Millimeter-Wave Open-Resonator Techniques," in *Infrared and Millimeter Waves,* vol. 10, ch. 4, New York: Academic Press, pp. 233–281.

[5] M. N. Afsar, X. Li, and H. Chi, "An automated 60 GHz open resonator system for precision dielectric measurements," *IEEE Trans. Microwave Theory Tech.,* vol. 38, pp. 1845–1853, Dec. 1990.

[6] H. Kogelnik and T. Li, "Laser beams and resonators," *Proc. IEEE,* vol. 54, no. 10, Oct. 1966, pp. 1312–1329.

[7] L. A. Weinstein, *Open Resonators and Open Waveguides.* Colorado, The Golem Press, 1969.

[8] A. L. Cullen and P. K. Yu, "Complex source-point theory of the open resonator," *Proc. Royal Soc.,* A 366, pp. 155–171, 1979.

[9] A. L. Cullen, "Note on the radiation associated with the excitation of an open resonator," *Electron. Lett.,* vol. 6, pp. 243–244, Apr. 1970.

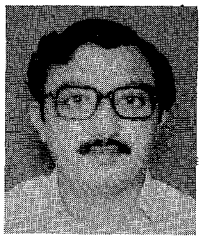
[10] R. E. Collin, *Field Theory Of Guided Waves,* 2 Ed., IEEE Press, 1991.

[11] C. G. Montgomery, R. H. Dicke and E. M. Pucell, Eds., *Principles of Microwave Circuits,* MIT Radiation Lab Series, New York: McGraw-Hill, 1948, p. 239.

[12] R. E. Collin, *Foundations for Microwave Engineering.* New York: McGraw Hill, 1966.

[13] S. B. Cohn, "Microwave coupling by large apertures," *Proc. IRE,* vol. 40, pp. 696–699, June 1952.

[14] Noel A. McDonald, "Electric and magnetic coupling through small apertures in shield walls of any thickness," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-20, pp. 689–695, Oct. 1972.



R. K. Mongia (S'87–M'88) obtained the degree of B.Sc. in Electrical Engineering from Delhi College of Engineering, Delhi, India, in 1981 and later, the Ph.D. degree from Indian Institute of Technology, Delhi, in 1989.

He also worked as a senior scientific officer-II in Centre for Applied Research in Electronics (CARE), IIT, Delhi from 1983–1990. During his stay at CARE, he worked on many research and development projects in the area of microwave and millimeter wave engineering. During 1990–91, he was a Research Associate at FAMU/FSU College of Engineering, Tallahassee, FL, where he worked on open resonators in the W-band frequency range. Presently, he is a post-doctoral fellow in the department of electrical engineering at the University of Ottawa, Ottawa, ON, Canada.

Rajendra K. Arora (S'64–M'65–SM'74) received a B.S. (Honors) degree in Physics from the University of Delhi, St. Stephen's College in 1956; a Diploma of the Indian Institute of Science, Bangalore,in electrical communication engineering, in 1959; and a Ph.D. degree in electrical engineering from the University of St. Andrews, Queen's College, Dundee, U.K., in 1965.

He joined the faculty of the University of Roorkee in 1959, studied in Britain on a Commonwealth Scholarship for the Ph.D. degree (1962–65), and returned to Roorkee where he was appointed a Professor in 1969. In 1972 he joined the Indian Institute of Technology Delhi as a Professor. He served there as the Dean of Students (1976–77) and as Head of the Department of Electrical Engineering (1982–84). From 1984 to 1986 he was a Visiting Professor with Clemson University, South Carolina, before he moved to his present position of Professor at the FAMU/FSU College of Engineering in 1987. He spent a year (1977–78) as a National Research Council Visiting Scientist at the National Oceanic and Atmospheric Administration, Environmental Research Laboratories, Boulder, Colorado; and has held short visiting research positions at the University of California, Santa Barbara; the Université de Rennes, France; the University College London, U.K.; and Allison Gas Turbine Division, General Motors, Indianapolis, Indiana. His research interests include microwaves, millimeter waves, phased array antennas, VLSI interconnect, quasi-optics, radar, electromagnetic scattering, high-temperature superconductivity and fiber-optics. He has authored about 80 research papers and has been the Principal Investigator of several sponsored research projects in these areas.

He is a recipient of the IEEE Centennial Medal, the Florida State University Excellence in Teaching Award, the (highest) University of Roorkee Khosla Research Award (Gold medal and a monetary award), Khosla Research Silver Medal, and other research prizes. In the All-India Engineering Services (Electronics) Examination held by the Union Public Service Commission in 1961, he obtained the First rank. He is a past Chairman of the IEEE Delhi Section.